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THEORETICAL INVESTIGATION OF THE EFFECT OF THE AILERONS ON THE WING OF AN AIRPLANE

By C. Wieselsberger

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THEORETICAL INVESTIGATION OF THE EFFECT OF THE AILERONS ON THE WING OF AN AIRPLANE.*

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If an airplane wing is moving in a straight line, and if both ailerons are deflected in opposite directions at a given instant, a torque is produced about the longitudinal axis of the airplane, due to the resulting unsymmetrical lift. As a result of the unequal induced drag of the two halves of the wing, a moment is simultaneously produced about the vertical axis, which we shall call the "moment of yaw." An accurate knowledge of the changed conditions resulting from the deflection of the ailerons, especially the knowledge of the absolute magnitude of the resulting moments, is important for airplane constructors in two respects. On the one hand, both these moments intimately affect the controllability of the airplane. The time required for the airplane to make a turn depends, in a high degree, on these moments. The knowledge of their magnitudes therefore supplies the basis for determining the turning ability. On the other hand, it is known that considerably higher stresses are produced in certain evolutions than in normal flight. The rotation of an airplane 360° about its longitudinal axis (rolling)

*From Report of the Aeronautical Research Institute, Tokyo Imperial University, No. 30, December, 1927, Vol. II, 16. pp. 421-446.

is an example of such a motion. It is initiated by the ailerons and increases the stresses in the wing spars. A quantitative knowledge of the torque is indispensable for a reliable strength calculation.

The present work investigates, on the basis of Prandtl's wing theory,* the form of the lift distribution when the ailerons are deflected in opposite directions. An ideal fluid and a wing with a rectangular plan form are assumed. The moments must not cause any rotation of the wing or any deviation from the rectilinear motion. We therefore consider these moments offset by suitable countermoments, so that the wing constantly maintains its original position, despite the deflection of the ailerons. After the lift distribution has been determined, the torque, additional induced drag and moment of yaw can be calculated. In Part II the conditions will be investigated for the case when both ailerons are deflected in the same direction.

With a normal aileron the trailing edge of the wing is removed and the lift at the wing ends is either increased or reduced by the action of the aileron. For a given deflection of the aileron, the foreshortened cross section may be regarded as a new wing profile whose angle of attack and camber have been changed with respect to the original profile. If the aerodynamic characteristics of the new profile are known (especially the in-

*L. Prandtl, "Tragflügeltheorie," I Mitteilung, Nachr. d. K. Gesellschaft d. Wissensch. zu Göttingen, Math. physik. Klasse 1918. L. Prandtl and A. Betz, "Vier Abhandlungen zur Hydrodynamik und Aerodynamik," Berlin, 1927 (J. Springer, Pub.).

clination of the lift curve to the axis of the angle of attack and the angle of attack at which the lift vanishes), the following investigations then apply exactly to actual conditions. A knowledge of the lift curve and angle of attack for zero lift is also desirable for the profile of the middle portion of the wing.

General Calculation Principles*

We will let b represent the wing span, x the distance of a point from the middle, and replace the coordinate x by the angle ϑ with the aid of the formula

$$x = -\frac{b}{2}\cos\vartheta \tag{1}$$

so that the angle ϑ varies from 0 to π, as x passes from one wing tip to the other. On the introduction of the angle ϑ as

*The method on which the following calculation is based, is taken essentially from E. Trefftz. See E. Trefftz, "Prandtl'sche Tragflächen- und Propeller-Theorie," Z. f. angew. Math. u. Mech. 1921, p.206. In a somewhat modified form, it is also found in H. Glauert's book, "The Elements of Aerofoil and Airscrew Theory," Cambridge, 1926. This calculation method, which is given here with a slight supplementation, can be used to advantage for solving a series of propeller problems.

The following articles constitute valuable contributions to the questions treated here, without attempting a direct quantitative solution.

Max M. Munk, "A New Relation between the Induced Yawing Moment and the Rolling Moment of an Airfoil in Straight Motion," N.A.C.A. Technical Report No. 197 (incorporated in Tenth Annual Report, 1924). Munk gives a general relation between the rolling moment and the yawing moment, without determining the lift distribution and the magnitude of the moments themselves.

F. N. Scheubel, "Quermoment und Kursmoment eines Tragflügels im geraden Flug," Z.F.M. 1925, p.152.

J. B. Scarborough, "Some Problems on the Lift and Rolling Moment of Airplane Wings," N.A.C.A. Technical Report No. 200 (Tenth Annual Report, 1924).

a variable, a point in the span is therefore determined not by its distance from the center, but by the angle ϑ. If Γ denotes the circulation and v the flight speed, any desired distribution of the circulation over the span can be represented by a Fourier series of the form

$$\Gamma = 2bv \sum_{n=1}^{\infty} a_n \sin n\vartheta \qquad (2)$$

whereby, for a wing of a given shape, the coefficient a_n must be so determined that they accord with the conditions of the wing theory. One of these conditions requires the geometric angle of attack α to be equivalent, at every point of the span, to the sum of the actual angle of attack α' and the induced angle of attack w/v. w denotes the induced vertical velocity at the position of the wing and will henceforth be designated as the "downflow." We thus obtain the conditional equation

$$\alpha' + \frac{w}{v} = \alpha \qquad (3)$$

The downflow w is also a function of the circulation distribution. At any given position x_o it is expressed by the circulation as follows

$$w(x_o) = \frac{1}{4\pi} \int_{-\frac{b}{2}}^{+\frac{b}{2}} \frac{d\Gamma}{dx} \frac{dx}{x - x_o} .$$

(See L. Prandtl, "Tragflügeltheorie" I, p.19.) By introducing our new variable ϑ instead of x, the downflow at the point ϑ_o becomes

$$w(\vartheta_o) = \frac{v}{\pi} \int_0^{\pi} \frac{\Sigma\, n\, a_n \cos n\vartheta}{\cos\vartheta - \cos\vartheta_o} d\vartheta$$

The integration of the expression

$$l = \int_0^\pi \frac{\cos n\vartheta}{\cos\vartheta - \cos\vartheta_0} d\vartheta$$

gives the "main value"

$$l = \pi \frac{\sin n\vartheta_0}{\sin\vartheta_0} .$$

The downflow then becomes

$$w(\vartheta_0) = v \Sigma n a_n \frac{\sin n\vartheta_0}{\sin\vartheta_0} \tag{4}$$

This expression is introduced into the conditional equation (3), after the effective angle of attack α' has been replaced by a suitable expression. The angle of attack is always measured from that position of the wing at which the lift vanishes. Then the lift coefficient c_a is directly proportional to the effective angle of attack,

$$c_a = 2c_1 \alpha' \tag{5}$$

For an infinite span and for the customary profiles, the value of the constant c_1 approaches that of π (α' in circular measure), which value will be used in the following calculations. The calculations can be made in like manner for any other value of c_1, should it seem desirable (See remark at the end).

With t as the wing chord, the circulation, on the basis of the Kutta-Joukowsky formula, can be written

$$\Gamma = \frac{c_a}{2} vt$$

and by using equation (5), we obtain

$$\alpha' = \frac{\Gamma}{c_1\, vt}.$$

If we introduce this value of α' into the conditional equation (3) and simultaneously replace Γ by its equivalent expression in equation (2), and the downflow by equation (4) (in which we now write simply ϑ instead of ϑ_0) we obtain, for further calculations, the fundamental equation

$$\sum_{n=1}^{\infty} a_n \sin n\vartheta \left(n + \frac{2b}{c_1\, t} \sin\vartheta\right) = \alpha \sin\vartheta \qquad (6)$$

This equation contains no unknown quantities aside from the Fourier coefficients a_n. It must be satisfied at every point between $\vartheta = 0$ and π. Since we must naturally be limited to a finite number of coefficients a_n, this means that the fundamental equation can be satisfied at only a finite number of points. If we introduce into the fundamental equation, the values $\vartheta_1, \vartheta_2, \vartheta_3 \ldots\ldots \vartheta_i$, which represent different points of the span with the angles of attack $\alpha_1, \alpha_2, \alpha_3 \ldots\ldots \alpha_i$, and retain the first i terms of the Fourier series, we thus obtain i linear equations with i unknowns $a_1, a_2, a_3 \ldots\ldots a_i$, which can be calculated by known methods.

In our case, we have a rectangular wing whose angle of attack varies irregularly. We must therefore introduce into our system of equations, for points in the middle portion of the wing, the angle of attack existing at those points and, for points within the span of the ailerons, the enlarged or reduced angle of attack. The conditions at the wing tips require spe-

cial attention. According to equation (2), the circulation at the wing tips ($\vartheta = 0$ and π) assumes the value zero. If we now make up the system of equations for several points of the span, without including the ends, and calculate the distribution of the circulation, we find that, in our case, the circulation at the ends goes through to zero in a very peculiar manner, namely through the negative region and, further, that the induced downflow assumes useless values at the ends. These disagreements are removed by also including the ends in the calculation. For this purpose, we must write the fundamental equation (6) for the limiting case $\vartheta = 0$ (and π)

$$\lim_{\vartheta=0}\left[\Sigma\ a_n \sin n\vartheta \left(n + \frac{2b}{c_1 t} \sin\vartheta\right) = \alpha \sin\vartheta\right]$$

This limiting case yields the equation

$$\Sigma\ n^2\ a_n = \alpha$$

and consequently, if we write $\frac{2b}{c_1 t} = p$ for short, we arrive at the following system of equations for calculating the unknown coefficients $a_1, a_2, \ldots\ldots a_i$:

$$\left.\begin{array}{l}
a_1 + 4a_2 + 9a_3 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots + i\ a_i = \vartheta_0 \\
a_i \sin\vartheta_1 (1 + p \sin\vartheta_1) + a_2 \sin 2\vartheta_1 (2 + p \sin\vartheta_1) + \ldots + a_i \sin i\vartheta_1 (i + p \sin\vartheta_1) \\
\qquad = \alpha_1 \sin\vartheta_1 \\
\vdots \qquad\qquad \vdots \qquad\qquad \vdots \\
a_1 \sin\vartheta_i (1 + p \sin\vartheta_i) + a_2 \sin 2\vartheta_i (2 + p \sin\vartheta_i) + \ldots + a_i \sin i\vartheta_i (i + p \sin\vartheta_i) \\
\qquad = \alpha_i \sin\vartheta_i
\end{array}\right\} (7)$$

With the aid of this system, it is possible to calculate wings of any plan form and twist or warping for any lift law, which can itself vary from point to point, provided only that the "lifting line" is a straight line and is perpendicular to the direction of flight.

Part I. Ailerons Deflected in Opposite Directions

Calculation of the Circulation Distribution

In what follows, α denotes the angle of attack of the middle portion of the wing. The angle of attack of the aileron portions of the wing are designated by α_r for one end and by α_l for the other end. The differences $\alpha - \alpha_r$ and $\alpha - \alpha_l$ are then distinguished only by the sign, since the ailerons are deflected to the same degree, but in opposite directions. We shall further assume that the forces generated by the ailerons are equal and are distinguished only by their signs. The difference in the angles of attack is denoted by α_q. Hence,

$$|\alpha - \alpha_r| = |\alpha - \alpha_l| = \alpha_q .$$

This angle α_q is not identical with the angle which the wing chords form with one another, since, according to our arrangement, the angles of attack must be measured from the position of zero lift, and this position is not the same, due to the different profile shape. The value of the effective angle-of-attack difference α_q can be easily determined for a given case, when the profile positions of zero angle of attack are known.

Our problem can be still further simplified, if we imagine the resulting distribution of the circulation to be composed of two parts, namely, the circulation for a wing with the constant angle of attack α over the whole span and the circulation for a wing with zero angle of attack in its middle portion and an angle of attack of $+\alpha_q$ or $-\alpha_q$ in the end portions corresponding to the length of the ailerons. For the first component (with a constant angle of attack over the whole span and a rectangular contour) we can use the calculations of A. Betz, R. Fuchs, and H. Glauert.* Here the distribution of the circulation is symmetrical with relation to the middle portion of the wing and may be expressed by the odd terms of the Fourier series (equation (2)):

$$\Gamma_1 = 2bv\,(a_1 \sin\vartheta + a_3 \sin 3\vartheta + a_5 \sin 5\vartheta + \ldots\ldots)$$

The second component which contains the aileron effect, yields a symmetrical distribution of the circulation and is represented by the even terms

$$\Gamma_2 = 2bv\,(a_2 \sin 2\vartheta + a_4 \sin 4\vartheta + a \sin 6\vartheta + \ldots\ldots)$$

We can therefore limit ourselves to the calculation of this second part and retain only the even terms in the system of

*A. Betz, "Beiträge zur Tragflügeltheorie mit besonderer Berücksichtigung des einfachen rechteckigen Flügels," published in Berichte und Abhandlungen der Wissenschaftlichen Gesellschaft für Luftfahrt, 1920, No. 2.
R. Fuchs and L. Hopf, "Aerodynamik," Berlin, 1922.

equations (7). The fundamental equation (6) also contains the quantity $\frac{2b}{c_1 t}$ as parameter, and we must decide for which value to make the calculation. As regards c_1 we have already determined the value $c_1 = \pi$. For the aspect ratio b/t, we will adopt the value 2π, so that $\frac{2b}{c_1 t} = 4$. As regards the angle of attack, we have zero for the middle portion and α_q or $-\alpha_q$ for the ends, though it appears unnecessary to settle in advance on any definite value. The coefficients of the Fourier series appear as linear functions of α_q. Moreover, since the circulation values at corresponding points on the two half-wings are distinguished by the signs, it is only necessary to make the calculation for one-half, i.e., for ϑ between 0 and $\pi/2$. On the other hand, it must now be determined at what point the value of the angle of attack passes from α_q to 0. The numerical calculation was carried out for eight points of a half-wing* for the values $\vartheta = 0, 20, 35, 45, 55, 65, 75$ and 85°, and the corresponding first eight even Fourier coefficients $a_2, a_4, \ldots\ldots a_{16}$ were retained. The point of the change in the angle of attack was then shifted to between two consecutive values of this angle in the vicinity of the points

ϑ =	40°	60°	70°	90°
$2l/b$ =	0.234	0.5	0.658	1.0

whereby $2l/b$ is the ratio of the aileron length l to the half-spam $b/2$. When we have determined the transition point in this way, we do not then really have the case of a sudden indefinite

*A calculation of the distribution for six points of a half-wing was still unsatisfactory.

change in the angle of attack, but we have only confined the transition region between two limits. These limits can be made as narrow as desired, but this means a refinement of the conditions and would result in still higher coefficients of the Fourier series having to be accepted. Here, however, we will be content with the determination of the transition point between the above-mentioned limits, since it is accurate enough for all practical purposes. This method, moreover, has the mathematical advantage that the four cases can be calculated with like values of ϑ for ailerons of different lengths, which fact greatly simplifies the numerical calculation. The left sides of the system of equation (7), for example, remain unchanged when the transition point is shifted to between two other values of ϑ. After we have thus solved the system of equations for a given case, we can then very easily obtain the solution for another position of the transition point by simply calculating anew the right sides of the system. This calculation is very simple and short, while the solution of the whole system requires considerable time and effort. The numerical calculation was made accurately to four decimal places. For the four cases with ailerons of different lengths, the ratio a_{2n}/α_q of the Fourier coefficients a_{2n} to the angle of attack α_q gave the following values.

$2l/b$	ϑ	a_2/α_q	a_4/α_q	a_6/α_q	a_8/α_q
0.234	40°	0.0601	0.0461	0.0099	-0.0112
0.500	60°	0.1276	0.0324	-0.0183	0.0035
0.658	70°	0.1456	0.0035	-0.0046	0.0210
1.000	90°	0.1715	-0.0292	0.0287	-0.0061

$2l/b$	ϑ	a_{10}/α_q	a_{12}/α_q	a_{14}/α_q	a_{16}/α_q
0.234	40°	-0.0097	0.0005	0.0046	0.0019
0.500	60°	-0.0010	-0.0098	0.0034	0.0058
0.658	70°	-0.0012	0.0083	0.0012	-0.0072
1.000	90°	0.0193	-0.0036	0.0069	-0.0089

The sums $\Sigma \frac{a_{2n}}{\alpha_q} \sin 2n\vartheta$ calculated from these coefficients at the adopted points ϑ are given in the following table.

$2l/b$	$\vartheta = 20°$	35°	45°	55°	65°	75°	85°
0.234	0.0859	0.0971	0.0358	0.0163	0.0087	0.0043	0.0005
0.500	0.1010	0.1388	0.1415	0.1250	0.0448	0.0172	0.0048
0.658	0.0969	0.1320	0.1477	0.1495	0.1269	0.0396	0.0079
1.000	0.0997	0.1360	0.1553	0.1615	0.1576	0.1435	0.0912

The calculated circulation distributions for a half-wing are represented graphically in Figure 1. The circulation distribution is also plotted (dash line) for a rectangular wing with the aspect ratio 2π and with a constant angle of attack α_q

over the whole span. For this purpose, the results of Glauert's calculations were used, which showed that the symmetrical distribution is represented with sufficient accuracy for a simple wing by the first four coefficients of the Fourier series. Glauert adopted, for his calculation, the following four values of ϑ :

$$\vartheta = 22.5^\circ, \quad 45^\circ, \quad 67.5^\circ, \quad 90^\circ.$$

The coefficients $\frac{a_{2n-1}}{\alpha_q}$ and the sums $\Sigma \frac{a_{2n-1}}{\alpha_q} \sin(2n-1)\vartheta$ gave the following results:

n=	1	3	5	7
$\frac{a_{2n-1}}{\alpha_q}$	0.232	0.0287	0.0057	0.0010
$\Sigma \frac{a_{2n-1}}{\alpha_q} \sin(2n-1)\vartheta$	0.1210	0.1796	0.2021	0.2080

The graphic representation shows that the circulation drops off variously toward the middle to zero according to the length of the aileron. Even the central portion, with zero angle of attack, receives some circulation, due to the induced vertical velocity.

Moments and Induced Drag

We can now easily calculate the rolling moment M_y of the wing with the aid of the known distribution of the circulation. The lift dA of a wing element has the value

$$dA = \rho\, v\, \Gamma\, dx$$

and the rolling moment of the whole wing is therefore

$$M_y = \int_{-\frac{b}{2}}^{+\frac{b}{2}} \rho\, v\, \Gamma\, x dx = -\frac{\rho v^2}{2} \frac{b^3}{2} \int_0^{\pi} \Sigma\, a_{2n} \sin 2n\vartheta \sin 2\vartheta\, d\vartheta$$

$$= -\frac{\rho v^2}{2}\, b^3\, \frac{\pi}{4}\, a_2$$

whereby it is worth noting that only the coefficient a_2 affects the rolling moment. Since a_2 is expressed by the angle α_q, we may write

$$M_y = -\frac{\rho v^2}{2}\, b^3\, \zeta\, \alpha_q \qquad (8)$$

whereby

$$\zeta = \frac{\pi}{4} \frac{a_2}{\alpha_q}$$

and has the following values for different aileron lengths.

$2l/b$	0.234	0.500	0.658	1.000
ζ	0.047	0.100	0.114	0.135

In Figure 2 the determining quantities ϑ for the rolling moment are plotted against the ratio $2l/b$ and connected by a curve starting from the origin. With the aid of this diagram, we are enabled to calculate the rolling moment for any aileron length by introducing the coefficients ϑ into equation (8).

Our next task is to calculate the additional induced drag produced by the aileron deflection. The induced drag of the wing is increased, because the distribution of the circulation is rendered unsymmetrical by the aileron deflection and therefore differs considerably from the best distribution, which is elliptical. If we also imagine the middle of the wing as having the angle α, the distribution of the circulation is then represented by both the even and odd terms of the Fourier series. The total induced drag W_i then becomes

$$W_i = \int_{-\frac{b}{2}}^{+\frac{b}{2}} \frac{w}{V}\, dA = \frac{\rho V^2}{2} 2b^2 \int_0^{\pi} \Sigma\, n\, a_n \sin n\vartheta\, \Sigma\, a_n \sin n\vartheta\, d\vartheta$$

$$= \frac{\rho V^2}{2} 2b^2 \int_0^{\pi} (a_1 \sin\vartheta + 2a_2 \sin 2\vartheta + 3a_3 \sin 3\vartheta + \ldots\ldots)(a_1 \sin\vartheta + a_2 \sin 2\vartheta + a_3 \sin 3\vartheta + \ldots\ldots) d\vartheta$$

If the integrant is then multiplied term by term, it is found that the terms with the mixed coefficients yield no contribution to the integral. There remains only

$$W_i = \frac{\rho V^2}{2} 2b^2 \int_0^{\pi} (a_1^2 \sin^2\vartheta + 2\, a_2^2 \sin^2 2\vartheta + 3\, a_3^2 \sin^2 3\vartheta + \ldots\ldots) d\vartheta$$

$$= \frac{\rho V^2}{2} 2b^2\, \pi\, (a_1^2 + 2a_2^2 + 3a_3^2 + \ldots\ldots)$$

or

$$W_i = \frac{\rho V^2}{2} b^2\, \pi\, \Sigma\, n\, a_n^2 .$$

We can also write this equation in the form

$$W_i = \frac{\rho V^2}{2} b^2\, \pi \left(\Sigma (2n - 1)\, a_{2n-1}^2 + \Sigma\, 2n\, a_{2n}^2\right)$$

i.e., we add the even and odd coefficients separately and then add the two partial sums. We now see that the sum of the odd terms comes from the symmetrical distribution, while the even terms represent that part of the drag produced by the unsymmetrical distribution, i.e., by the aileron deflection. The additional induced drag W_{iq} is therefore

$$W_{iq} = \frac{\rho v^2}{2} b^2 \pi \Sigma 2n\, a_{2n}^2 \qquad (9)$$

Since the coefficients a_{2n} are proportional to the angle α_q, it is obvious that the additional drag increases as the square of the angle α_q. If we put

$$\pi \Sigma 2n \left(\frac{a_{2n}}{\alpha_q}\right)^2 = \eta$$

we may then also write

$$W_{iq} = \frac{\rho v^2}{2} b^2 \eta \alpha_q^2$$

whereby the factor η of the additional induced drag has the following values for different aileron lengths:

$2l/b$	0.234	0.500	0.658	1.00
η	0.0462	0.1114	0.1440	0.1980

In Figure 2, η is plotted against $2l/b$, thus enabling us also to calculate the additional induced drag for any desired aileron length.

Lastly, we also calculate the indicated moment of yaw M_z about the vertical axis by multiplying the induced drag of a wing element by the distance x from the center of the wing and integrating it over the span:

$$M_z = \int_{-\frac{b}{2}}^{+\frac{b}{2}} x \frac{w}{v} dA = \frac{\rho v^2}{2} b^3 \int_0^\pi \cos\vartheta \, \Sigma\, n\, a_n \sin n\vartheta \, \Sigma\, a_n \sin n\vartheta \, d\vartheta$$

$$= \frac{\rho v^2}{2} b^3 \int_0^\pi \cos\vartheta \, (a_1 \sin\vartheta + 2a_2 \sin 2\vartheta + 3a_3 \sin 3\vartheta + \ldots\ldots)$$
$$(a_1 \sin\vartheta + a_2 \sin 2\vartheta + a_3 \sin 3\vartheta + \ldots\ldots) d\vartheta$$

If we multiply the integrant term by term, we thus obtain partial integrals of the form

$$\int_0^\pi a_m\, a_n \cos\vartheta \sin m\vartheta \sin n\vartheta \, d\vartheta$$

Between the limits 0 and π, all the integrals disappear with the exception of those having the form

$$\int_0^\pi a_{n+1}\, a_n \cos\vartheta \sin (n + 1)\, \vartheta \sin n\vartheta \, d\vartheta$$

in which the indices differ only by unity. Their evaluation gives

$$M_z = \frac{\rho v^2}{2}\, b^3\, \frac{\pi}{4}\, (3a_1\; a_2 + 5a_2\; a_3 + 7a_3\; a_4 + \ldots\ldots)$$

or

$$M_z = \frac{\rho v^2}{2}\, b^3\, \frac{\pi}{4}\, \Sigma\, (2n + 1)\; a_n\; a_{n+1}$$

It is obvious that the coefficients a_1, a_3, a_5, ... of the symmetrical distribution also affect the moment of yaw. If, in a manner analogous to the above, we again write

$$\frac{\pi}{4}\, \Sigma\, (2n + 1)\, \frac{a_n}{\alpha_q}\, \frac{a_{n+1}}{\alpha} = \xi$$

the moment of yaw becomes

$$M_z = \frac{\rho v^2}{2}\, b^3\, \xi\; \alpha\; \alpha_q. \qquad (10)$$

The calculation yields the following values for the factor ξ:

$2l/b$	0.234	0.500	0.658	1.000
ξ	0.0492	0.0895	0.0965	0.1089

These values are also represented graphically in Figure 2.

For the case when the coefficients of the symmetrical distribution, with the exception of the coefficients a_1, are all put at zero, we bbtain the elliptical distribution.

$$\Gamma = 2\ b\ v\ a_1 \sin\vartheta$$

which is known to play an important role in the wing theory. As already mentioned, Munk suggested, for this distribution, a relatiom between the moments of roll and yaw, which we can now verify. In this case the moment of yaw is

$$M_z = \frac{\rho v^2}{2} b^3 \frac{\pi}{4} 3 a_1 a_2 .$$

If we calculate the lift for the elliptical distribution, we can express the coefficient a_1 by the coefficient of lift c_a. We will omit this simple calculation and only indicate the result

$$a_1 = \frac{c_a F}{\pi b^2} ,$$

Consequently the moment of yaw becomes

$$M_z = \frac{\rho v^2}{2} b^3 \frac{3 c_a F}{4 b^2} a_2$$

By using the expression for the rolling moment (equation (8)), we find the ratio of the two moments

$$\frac{M_z}{M_y} = \frac{3}{\pi} c_a \frac{F}{b^2}$$

which accords with Munk's formula.

Part II. Ailerons Deflected in Same Direction

Although the case when both ailerons are deflected in the same direction is not so important as the one just considered, nevertheless a knowledge of the changed relations is sometimes of interest. It has been proposed to reduce the landing speed by deflecting both ailerons downward, thus increasing both the lift and the drag. A quantitative knowledge of these forces is therefore desirable for determining the effectiveness of such a device.

Calculation of the Circulation Distribution

This calculation was based on the same assumptions as in Part I. In particular, the same aspect ratio $b/t = 2\pi$ and the constant $c_1 = \pi$ were adopted. Contrary to Part I, and due to the deflection of both ailerons in the same direction, the distribution of the circulation is symmetrical with respect to the middle of the wing. We must therefore express the circulation distribution by the uneven coefficients of the Fourier series and write

$$\Gamma = 2bv\,(a_1 \sin\vartheta + a_3 \sin 3\vartheta + a_5 \sin 5\vartheta + \ldots\ldots).$$

The numerical calculation was repeated for eight points of a half-wing and for the same ϑ values as in Part I. The changes in the angle of attack were made in the vicinity of the following ϑ values:

ϑ =	40°	60°	70°	80°
$2l/b$ =	0.234	0.500	0.658	0.826

The following values were obtained for the ratio $\frac{a_{2n-1}}{\alpha_q}$ of the Fourier coefficients a_{2n-1} to the difference α_q in the angles of attack.

$2l/b$	ϑ	a_1/α_q	a_3/α_q	a_5/α_q	a_7/α_q
0.234	40°	0.0415	0.0589	0.0277	-0.0037
0.500	60°	0.1037	0.0883	-0.0079	-0.0079
0.658	70°	0.1407	0.0777	-0.0221	0.0149
0.826	80°	0.1870	0.0599	-0.0157	0.0177

$2l/b$	ϑ	a_9/α_q	a_{11}/α_q	a_{13}/α_q	a_{15}/α_q
0.234	40°	-0.0122	-0.0042	0.0037	0.0035
0.500	60°	0.0084	-0.0067	-0.0040	0.0057
0.658	70°	0.0035	-0.0027	0.0078	-0.0059
0.826	80°	-0.0124	0.0081	-0.0058	0.0034

The sums

$$\frac{a_{2n-1}}{\alpha_q} \sin (2n - 1) \vartheta$$

calculated with the aid of these coefficients are given in the following table.

$2l/b$	ϑ=20°	35°	45°	55°	65°	75°	85°
0.234	0.0861	0.0978	0.0374	0.0183	0.0115	0.0081	0.0070
0.500	0.1016	0.1402	0.1463	0.1311	0.0528	0.0299	0.0238
0.658	0.1025	0.1429	0.1588	0.1623	0.1448	0.0622	0.0418
0.826	0.1093	0.1550	0.1718	0.1794	0.1806	0.1634	0.0829

Figure 3 shows the distribution of the circulation over the half-span for different aileron lengths, whereby the distribution for the rectangular wing, with constant angle α_q over the whole span, is plotted as a dash line. With deflected ailerons the middle portion of the wing with zero angle of attack (due to the generated upward current) likewise acquires lift in proportion to the length of the ailerons.

Lift and Drag

Due to the symmetrical distribution of the circulation, no moments are generated and we have to calculate only the lift and induced drag. For the wing lift, we have

$$A = \int_{-\frac{b}{2}}^{+\frac{b}{2}} \rho\, v\, \Gamma\, dx$$

or, if we introduce the angle ϑ,

$$A = \rho\, v^2\, b^2 \int_0^{\pi} \Sigma a_{2n-1} \sin (2n - 1)\vartheta \sin\vartheta \, d\vartheta$$

The integration yields

$$A = \frac{\rho v^2}{2}\, b^2\, a_1 \pi \tag{11}$$

It is seen that the lift depends only on the coefficient a_1. The remaining coefficients affect the form of the lift distribution, but not the amount of the lift. Equation (11) can also be written in the form

$$A = \frac{\rho v^2}{2}\, b^2\, \lambda\, \alpha_q$$

whereby $\lambda = \pi \frac{a_1}{\alpha_q}$ and has the following values for different aileron lengths.

$2l/b$	6.234	0.500	0.658	0.826	1.000
λ	0.130	0.326	0.442	0.587	0.729

In Figure 4, λ is plotted as a function of $2l/b$. With the aid of this curve, we can calculate the lift for any desired aileron length.

For the induced drag, we obtain, similar to equation (9) of Part I

$$W_{iq} = \frac{\rho v^2}{2} b^2 \pi \Sigma (2n - 1) a_{2n-1}^2$$

or with

$$\pi \Sigma (2n - 1) \left(\frac{a_{2n-1}}{\alpha_q}\right) = \kappa$$

in which κ has the following values:

$2l/b$	0.234	0.500	0.658	0.826	1.000
κ	0.056	0.116	0.137	0.163	0.178

The value of κ as a function of $2l/b$ is also plotted in Figure 4.

Remark.- The above calculations could not be made to apply universally, as would be desirable, because we had to limit ourselves to certain values for the aspect ratio b/t and for the factor c_1. We adopted $b/t = 2\pi$ and $c_1 = \pi$, so that $2b/c_1 t = 4$ in the fundamental equation (6). Since the aspect ratio and the factor c_1 occur only in this combination in the fundamental equation, another value for the factor c_1 can be adopted without affecting the value of the quotient, provided the aspect ratio is changed correspondingly at the same time. In this case we naturally obtain the same values for the rolling moment, induced drag, etc. If, for example, we take $c_1 = 2.8$, we obtain the corresponding aspect ratio

$$\frac{b}{t} = 2c_1 = 5.6.$$

This shows that other values for b/t and c_1 yield quite different relations. On the other hand, it would be interesting to know what the result would be if the calculation were made for another c_1 without changing the aspect ratio. With this purpose in mind, we have calculated the lift distribution for the case $b/t = 2\pi$ and $c_1 = 2.51$, so that the quotient $2b/c_1t = 5$. Moreover, the deflection of the ailerons in the same direction and an aileron length of $2l/b = 0.5$ were assumed. The following values were found for the coefficients λ and κ, whereby the coefficients for the case $c_1 = \pi$, as likewise the quotients for the corresponding values, are given for comparison:

c_1	λ	κ
2.51	0.278	0.0887
π	0.326	0.116
0.8	0.853	0.764

From the quotients in the last line, we gather the following information.

If the factor c_1 is reduced 20% without changing the aspect ratio, the lift is reduced about 15% and the drag about 24%. Any accurate calculation requires therefore the solution of the system of equations (7) for the given values b/t and c_1. For a first rough correction, it suffices perhaps in many cases, where $c_1 \neq \pi$, to multiply the factors ζ, η, ξ, λ and κ, calculated for the case $c_1 = \pi$, by c_1/π. In the above case we would then have to accept an error of -6% in lift and 5% in drag, while the errors without this correction would be 17% and 31%, respectively. A similar correction can be made, even with a changed aspect ratio. The agreement will naturally be so much the better, the less the value of c_1 differs from π.

S u m m a r y

The effect of the ailerons is quantitatively investigated on the basis of Prandtl's wing theory. In Part I a simple rectangular wing is assumed, whose angle of attack, corresponding to the aileron deflection, is increased at one end and decreased by a like amount at the other end of the wing. The distribution of the circulation can be expressed by a Fourier series, whose coefficients are determined from a system of linear equations. The distribution is calculated for different aileron lengths. The rolling moment, the additional induced drag and the yawing moment can all be determined from the distribution. In Part II,

the relations are investigated for the case when both ailerons are deflected in the same direction, and the lift and induced drag are calculated for different aileron lengths.

Translation by Dwight M. Miner,
National Advisory Committee
for Aeronautics.

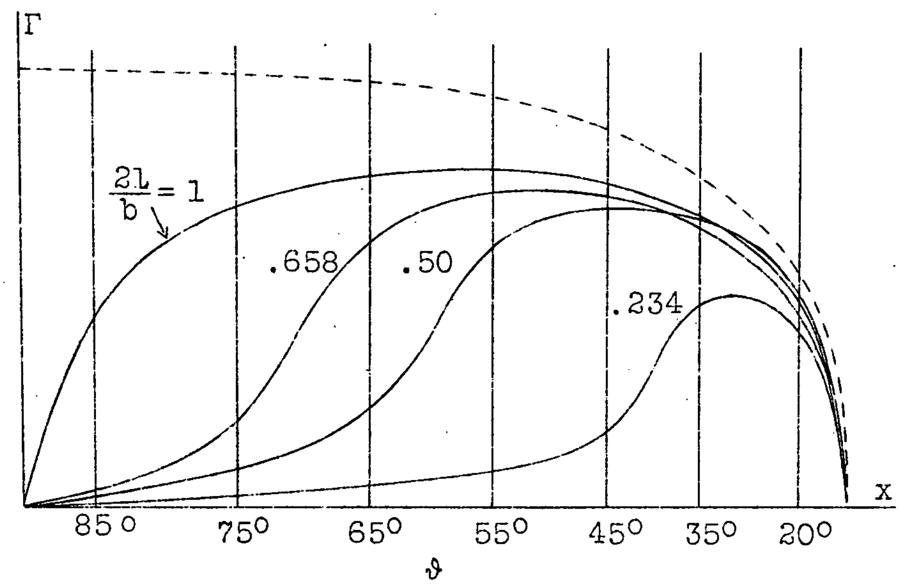
Γ
2l/b = 1
.658
.50
.234
x
85°
75°
65°
55°
45°
35°
20°
ϑ

Fig.1

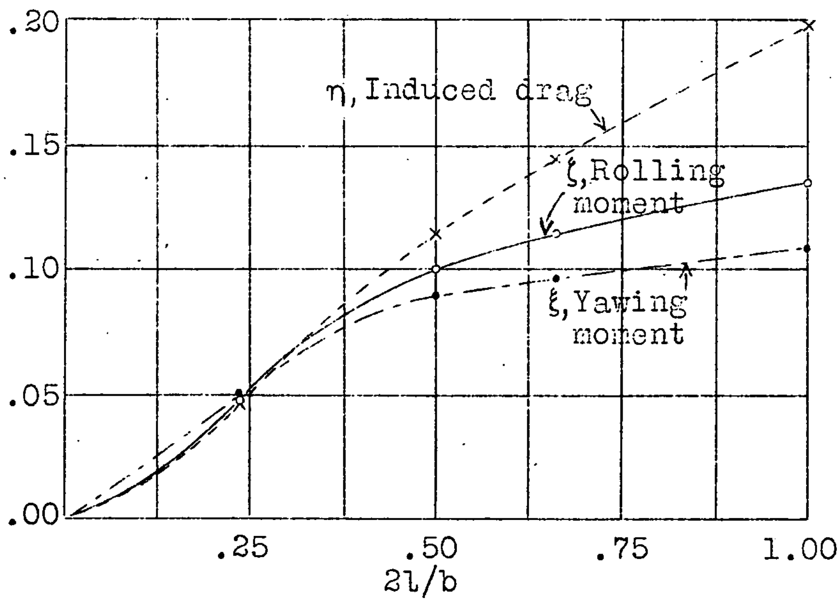
.20
.15
.10
.05
.00
η, Induced drag
ζ, Rolling moment
ξ, Yawing moment
.25
.50
.75
1.00
2l/b

Fig.2

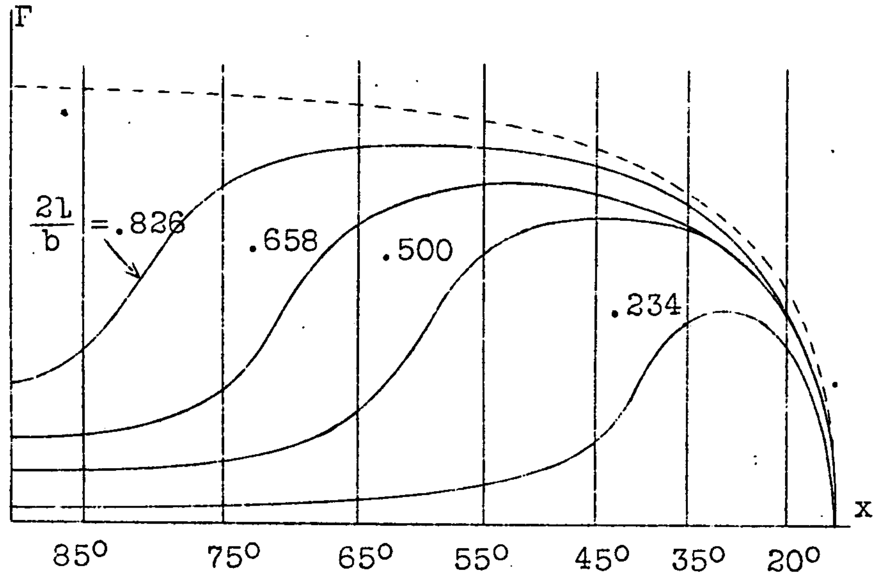
F
2l/b = .826
.658
.500
.234
x
85°
75°
65°
55°
45°
35°
20°
ϑ

Fig.3

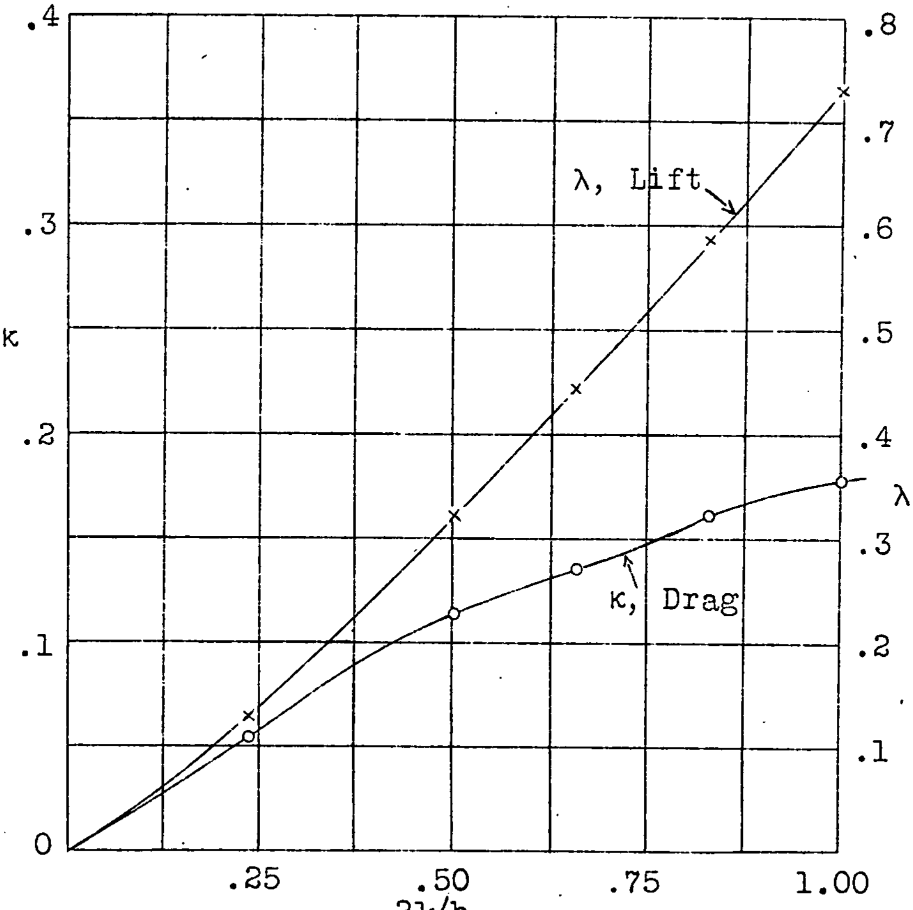
.4
.3
.2
.1
0
κ
.8
.7
.6
.5
.4
.3
.2
.1
λ
λ, Lift
κ, Drag
.25
.50
.75
1.00
2l/b

Fig.4